COMMONS, PRIVATISATION, AND THE DISTRIBUTION OF WEALTH IN TRADEABLE ASSETS

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Contents

1. Introduction

Herein I further an investigation into the insurance value of communally *vs* privately held capital by considering the distributional consequences of allowing trade in private capital holdings in the absence of both credit markets and contingent assets.[1] Baland and Francois [2005, hereafter *BF*] consider two plausible sources of incomplete markets in insurance against idiosyncratic (individual) fluctuations in labour productivity. When owned assets cannot be traded, they find that consumption welfare may necessarily be lower for some agents under a private property regime as compared with group ownership of a common pool resource (CPR), even when the privatisation occurs equitably. The imperfections in insurance in the stories discussed by *BF* arise as a result of asymmetric information in one case and limited enforceability of contracts in the other.

However, *BF* assume that under privatisation, ownership rights extend only to excludableexcludable use of the capital and thus the ability to accrue rents. The option to sell or buy capital assets is unavailable. In the present work, I consider relaxing this constraint to evaluate the insurance benefit of being able to sell one's holdings to fund consumption in time of need.

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[1]I am grateful to Patrick Francois for suggesting the topic and to Chris Bidner and Ken Jackson for useful discussion.

Without credit constraints, the ability to trade in capital assets would offer a form of perfect insurance in the context analysed by *BF*; such privatisation would then be *ex ante* Pareto preferred over communal ownership. Here, however, I consider an asset market that is limited by credit constraints. This represents a third form of imperfect insurance, to complement the two analysed by *BF.*

Within the given context, I seek to determine sufficient conditions for the privatisation of a CPR to be strictly welfare improving or reducing, given the availability of an asset market with credit constraints.

Below I outline in Section 2 the details of the CPR situation envisaged by *BF* and largely replicated in this work. Section 3 reviews the relevant literature on precautionary saving, Section 3 introduces a formal model, and Section 5 describes a numeric solution of a discretised version of the model.

2. Commons as insurance

This paper, like that of *BF*, is motivated by the observed resistance in rural, developing communities to adopt private management of resources in place of CPRs. *BF* review studies of commonly held property, typically low-productivity land or forest, which provides informal insurance against individual income shocks. For example, Pattanayak and Sills [2001] study collection trips made by rural agriculturalists into open-access forest. They show firstly that productivity in the forest is not well correlated with shocks to private agricultural returns and, secondly, that the poor make higher use of the CPR. This suggests that access to the CPR may play an important insurance role when there is individual uncertainty in an alternate, more productive sector.

Most compelling are studies in which private ownership and CPRs are simultaneously observed for different resources in a given community. In such cases, local participants are demonstrably aware of and able to implement institutions of private property and market mediated trade, yet those institutions are not used in the management of certain resources.

Ostrom [1990] provides evidence from a variety of instances of sustainable (persistent over 100 years) community management of resources by cultures which extensively use private property as well. She argues against the "tragedy" reflexively assumed to be inherent in attempts at CPR management and against the expectation that private property solves this tragedy for all non-fugitive (unlike *e.g.* fish or groundwater) resources. In order to point the way towards a theory accounting for such coexistence of communal and private management, Ostrom identifies properties of resources which tend to be managed as CPRs. Some of these point towards the story being addressed here; in particular Ostrom emphasises the prevalence in CPRs of resources with relatively low yield and often low improvability in comparison with other available sectors, and of the importance of income variance in motivating communal management.[2]

In the situation under consideration here and in *BF*, CPRs appear as fully open access resources in which there is no overwhelming "tragedy" of the commons because most participants have more productive options than working on the resource. Thus, even abstracting from the efficient communal management institutions such as self monitoring and enforcement found by Ostrom [1990], the commons may be a more efficient form of management than the realistically available options of incomplete private markets. The present study evaluates the trade-off between two flawed systems — the insurance value to risk averse agents of an over-used commons and the enhanced efficiency of privatised but incomplete markets.

3. Precautionary saving

A substantial literature exists on the role of precautionary saving in developed economies. In particular, a typical question has been the importance of individual income fluctuations in determining the aggregate savings level or the volume of asset trading. Standard growth models consider aggregate shocks to productivity but allow for no uninsurable idiosyncratic shocks. There is strong evidence in the U.S.A. that individual income fluctuations are large, even after taking into account the taxes, transfers, unemployment insurance, and support from family and friends which provide partial income insurance. Moreover, consumption responds strongly to uninsured income shocks (see references in Carroll [2000]).

The class of models dealing with this "income fluctuation problem" generally focus on assessing the effect of liquidity constraints. With uninsured risk in their labour endowment and with expected borrowing constraints in the future, agents accumulate "excess" capital in order to smooth consumption. Partially relaxing the credit constraint allows borrowing to substitute for precautionary saving, decreases the level of aggregate capital, and increases the interest rate towards the time preference rate.

If significant, this effect presents a problem for representative agent aggregation because it implies a consumption policy that is nonlinear in wealth. As a result, calculation of the full wealth distribution is central to evaluating the effect of credit constraints.

Aiyagari [1994] calculates numerical general equilibrium solutions for a standard, calibrated growth model modified to account for credit constraints and individually independent but serially-correlated shocks to labour income. The general conclusion from such studies is that consumers are able to

[2] This relates in part to a different channel for insurance, which amounts to an economy of scale property, also emphasised by Ostrom. If there is spatial variation in productivity of a resource, then communal ownership and management may be a feasible means to insuring against individual variance *within* the CPR resource.

accomplish considerable consumption smoothing and thus welfare enhancement by accumulating and decumulating assets in order to absorb income shocks. Nevertheless, for utility functions favoured by Aiyagari [1994], precautionary savings do not account for a large fraction of aggregate saving. While the credit constraints result in a large wealth inequality, it is not enough to match observed U.S. data.[3] Moreover, Carroll [2000] argues that despite the small aggregate correction to savings, other aggregate measures are profoundly different under a model with uninsurable idiosyncratic risk, putting welfare conclusions based on representative agent models in doubt.

Aiyagari [1994] concludes from his analysis that a non-zero tax on capital is likely to be optimal in the face of credit constraints. Such a tax could redistribute lump sum payments and thus partially insure against the individual uncertainty, while lowering the "excess" aggregate capital level.

Analogously, "tax" or insurance schemes (taxing labour income) were considered by *BF* as possible limited insurance systems under a privatised property regime in which sales of the resource itself were assumed not to be feasible. However, markets for selling asset holdings may be as realistic in their story as are insurance contracts.

Therefore, I consider the possibility of individual insurance through precautionary saving of privatised allocations of a fixed-size CPR. Like the literature outlined above, I abstract completely from aggregate shocks. Although it may be noted that the existence of aggregate shocks may worsen the externalities in communal management, increasing the overuse of a CPR, and is therefore relevant to the question at hand, incorporating aggregate shocks into a model of idiosyncratic shocks would entail a significant increase in computational complexity. In particular, the distribution of wealth would no longer be constant; it would evolve stochastically in response to the aggregate shocks and thus add a large additional state vector to the individual's problem.

A model of precautionary saving in the context of a partitioned and privatised CPR must differ somewhat from that of Aiyagari [1993, 1994]. I assume that the resource is, upon privatisation, initially partitioned equitably. In contrast to the Aiyagari model, however, the aggregate resource (or capital) amount is fixed; aggregate savings are assumed to be unfeasible.

A key difference between the question addressed by *BF* and that addressed here stems from the intertemporal nature of the investment decision. *BF* were able to compare the worst possible consumption levels between the two insurance environments — commons and private. In that case, if the poorest one-period realisation under private insurance is necessarily less desirable than the poorest one-period realisation under the commons, then there exists some set of (sufficiently risk averse) preferences for which privatisation would be Pareto inferior. On the other hand, when the equilibrium

[3]Cordoba [2004] discriminates between debt constraints and other market incompleteness in contributions to U.S. wealth inequality.

price of insurance is endogenously determined by the rate of intertemporal substitution, a neoclassical framework does not provide a welfare comparison based on one-period realisations. For instance, under a privatised insurance scheme, an infinitely risk averse individual with a minimum required consumption level $\underline{c}$ would never be forced below $\underline{c}$ if the initial wealth distribution was (equitable and) sufficient. That is, a rational individual would never sell enough assets to put herself at risk of starvation. This will be reflected in the equilibrium wealth distribution and price of the privatised resource. A sensible comparison between private and communal insurance therefore must consist of *ex-ante* expected welfare measures only.

4. Model

I model resource use under CPR management in the same way as described by *BF* and discussed below. I present a model of private resource trading that differs slightly from existing models of precautionary saving (for instance, see Aiyagari [1994]) in that the minimum individual labour income is endogenous and there is no aggregate saving.

Consider an economy populated with measure 1 agents, each endowed with an inelastic unit of labour per period. Agents receive each period an individual and independent draw of one-period private productivity θ according to the distribution function $f(\theta)$. The draws are independent across time and across individuals. Each agent may choose to allocate labour either to this private project, with return θ, or to a less productive constant returns to scale resource sector with total output $Y(K, L)$ and uniform labour productivity across workers. The aggregate quantity of resource K is fixed and there is no means by which aggregate saving is possible. The total labour L in the resource sector is determined endogenously.

In the case for which the resource is managed as a CPR, there is no intertemporal choice and allocations are as described by *BF*. Insurance is provided through the capture of the implicit resource rent, in addition to the labour rent, by any agent who chooses to work on the commons. In equilibrium all agents with realisations less than some threshold θ^c will choose to work on the commons.

Under private ownership, a different equilibrium threshold θ^P arises and determines who works in the resource sector. Such workers are hired competitively by resource owners who capture profit by selling Y, the numeraire, for consumption.[4] In addition, there is an asset market for trade of resource shares.

A steady state equilibrium for a given probability distribution $f(\theta)$ over individual one-period productivity realisations is a trade price p per unit of resource, a resource rent rate r, a wage w for work on the resource, a

[4] Equivalently, competitive firms rent the resource, hire labour, and sell output, making zero profit.

static one-period consumption policy $c(\theta, k)$, a saving policy $k^+(\theta, k)$, and a distribution of resource holdings $g(k)$ such that:

- given prices $\{p, r, w\}$, the policies $c(\cdot)$ and $k^+(\cdot)$ and labour choice threshold θ^P are solutions to the individual's dynamic optimisation problem for any individual realisation θ and for each possible asset level k. Formally, this dynamic problem is, for a given k_0 and any sequence of $\{\theta_t\}$,

$$\max_{\{c_t\},\{k_t^+\}} \mathbb{E}\left\langle \sum_{t=0}^{\infty} \beta^t u(c_t) \right\rangle$$

subject to

$$c_t + pk_t^+ = [1+r]pk_t + z^P(\theta_t) \quad \forall t$$

where $z^P(\theta_t) \equiv \max\{w, \theta_t\}$, and subject to the credit constraint[5]

$$k_t^+ \geq 0 \quad \forall t$$

- resource firms maximise their one-period rate of return by hiring labour and the resource:

$$\max_{K,L} F(K, L) - wL - rpK$$

- markets for labour clear:

$$L = \int_o^{\theta^P} f(\theta) d\theta$$

- markets for trade in the resource clear:

$$\int_0^{\infty}\int_0^{\infty} \left[k^+(\theta, k) - k\right] \, g(k)\, f(\theta)\, dk\, d\theta = 0 \tag{4.1}$$

- and the distribution of wealth and the aggregate resource level are constant:

$$\int_0^{\infty} k\, g(k)\, dk = K, \tag{4.2}$$

$$\int_0^{\infty}\int_0^{\infty} \mathbf{1}(k^+(\theta, k) - \hat{k})\, g(k)\, f(\theta)\, dk\, d\theta = g(\hat{k}), \quad \forall \hat{k} \tag{4.3}$$

In addition, the aggregate resource constraint will be satisfied given that other markets clear.

Equation (4.3) says that the measure of individuals ending up at any level of asset holdings $\hat{k}$ at the end of a period is equal to the measure $g(\hat{k})$ of individuals who began the period at that asset level. The function $\mathbf{1}(\cdot)$ is the indicator function; it takes value 1 when its argument is zero and takes value zero otherwise. This equation is awkwardly expressed. When the policy

[5]This constraint could be relaxed to a finite negative value; for instance, Aiyagari [1994] suggests the present value budget balance condition $pk^+ \geq -w/r$.

$k^+(\theta,k)$ is strictly monotonic[6] in k, the inverse function $\tilde{k}(\theta,k^+)$ exists. This inverse function gives the originating resource level for an individual receiving realisation θ and choosing new asset holding k^+. Then (4.3) can be written

$$\int_0^\infty g\left(\tilde{k}(\theta,k^+)\right) f(\theta)d\theta = g(k^+) \quad \forall k^+ \tag{4.4}$$

Proposition 4.1. *It follows from equation* (4.3) *that for equilibrium saving strategies that are monotonic in wealth, markets for trade in the resource clear each period (equation* (4.1)*) and the aggregate resource level is constant (equation* (4.2)*).*

Proof. Write the excess demand D for the resource as follows

$$D = \int_0^\infty\int_0^\infty \left[k^+(\theta,k) - k\right] g(k)\, f(\theta)\, dk\, d\theta$$

If the policy $k^+(\cdot)$ is strictly monotonic, it follows that $\int_0^\infty \mathbf{1}\left(k^+(\theta,k) - \hat{k}\right)\hat{k}\, d\hat{k} = k^+(\theta,k)$ since the integrand is zero for all but one value of $\hat{k}$. Splitting the expression for D above into its two terms and using this identity,

$$\begin{aligned} D &= \int_0^\infty\int_0^\infty \left[\int_0^\infty \mathbf{1}\left(k^+(\theta,k) - \hat{k}\right)\hat{k}\, d\hat{k}\right] g(k)\, f(\theta)\, dk\, d\theta \\ &\quad - \int_0^\infty\int_0^\infty k\, g(k)\, f(\theta)\, dk\, d\theta \\ &= \int_0^\infty \left[\int_0^\infty\int_0^\infty \mathbf{1}\left(k^+(\theta,k) - \hat{k}\right) g(k)\, f(\theta)\, dk\, d\theta\right]\hat{k}\, d\hat{k} \\ &\quad - \int_0^\infty k\, g(k)\, dk \end{aligned}$$

The above steps follow from switching the order of integration in the first term and from integration over θ in the second. Applying (4.3) in the first term gives

$$\begin{aligned} D &= \int_0^\infty \left[g(\hat{k})\right]\hat{k}\, d\hat{k} - \int_0^\infty k\, g(k)\, dk \\ &= 0 \end{aligned}$$

$\square$

Next I consider the value function approach to solving for equilibrium strategies. Let $V(k,\theta)$ be the optimal value function for an individual with

[6] There is no reason to expect *strict* monotonicity if, for instance, θ realisations are discretely distributed or there is a minimum subsistence level of consumption. In these instances agents' policy may be to sell all their assets (ending up with zero) for a continuous range of low asset levels.

resource holdings k and a current-period productivity draw θ; it solves the Bellman's equation

(4.5)

$$V(k,\theta) = \max_{k^+ \geq 0} \left\{ u\left(rpk + p[k - k^+] + z^P(\theta)\right) + \beta \int_0^\infty V(k^+,\theta^+) f(\theta^+) d\theta^+ \right\}$$

The associated Euler equation has the form

$$u_c(c) = \beta[1+r]\mathbb{E}\left\langle u_c(c^+)\right\rangle$$

where the expectation is taken over possible realisations of the next-period θ. There may be mass points at the extremes of the $g(\cdot)$ distribution.

Proposition 4.2. *The Markov process describing evolution of the economy is bounded; there is a minimum value of asset holdings and a maximum value of asset holdings. Furthermore, there exists a unique stationary distribution of asset holdings.*

Proof. See Aiyagari [1993]. □

Although few agents suffer binding credit constraints at any time, the inability to insure completely leads to a dispersion in wealth, even when initial allocations at the time of privatisation are equitable. At the time of privatisation, each agent foresees an *ex ante* expected lifetime utility under each of the alternative systems, CPR and private. Under privatisation, this expectation will reflect a complex set of possible transition paths towards a steady state wealth distribution and price. That is, the individual policy functions will evolve with the price towards the steady state. As a feasible means of comparing outcomes under the two systems, the lifetime expected utility of the *steady state* outcome under privatisation can be compared with that under insurance through the commons. Essentially, the question concerns the degree of wealth inequality incurred by privatisation.

5. Computable case

Due to the awkwardness of the equilibrium conditions, an analytic welfare comparison based on the general model above has not been found.[7] Proposition 4.2, however, suggests that a computation approach is possible. As a baseline case, I consider a simplified model with only two possible private labour productivities. Each agent has a probability λ of being lucky:

$$P(\theta) = \begin{cases} \lambda, & \theta = \theta_H \\ 1-\lambda, & \theta = \theta_L \end{cases}$$

where $\theta_H > \theta_L = 0$. Because each agent has an inelastic labour endowment, labour income is either θ_H or w each period. Normalising the aggregate

[7]In an appendix, I comment on a simple discrete case with upper and lower bounds on asset holdings. While some analytic progress is possible for this case, it is too specialised to address adequately the question of interest.

resource stock to 1, the aggregate output in the resource sector becomes $F(1, 1-\lambda)$.

By making θ_H high enough, I ensure that aggregate output in the resource sector is the same in the two management regimes; that is, as long as in the privatised equilibrium $\theta_L < w < \theta_H$, then regardless of the regime the population working in the resource sector is identically those agents in a low income state. This two-state model avoids the endogeneity of the real wage in the resource sector.

Under CPR management, income and thus consumption levels for high and low labour realisations are thus θ_H and $F(1, 1-\lambda)/[1-\lambda]$, respectively. Under private management, consumption levels for an agent with assets a are

$$c_H^P \equiv \theta_H + [1+r]a - a_H^+(a)$$

for the lucky and

$$c_L^P \equiv F_L(1, 1-\lambda) + [1+r]a - a_L^+(a)$$

for the unlucky. Here $a_H^+(a)$ and $a_L^+(a)$ are the policy functions, and resource holdings are expressed in terms of asset values. [8]

Under CPR management, the lifetime expected utility is

$$V_{\text{commons}} = \frac{\lambda u\left(\theta_H\right) + [1-\lambda]u\left(\frac{F(1,1-\lambda)}{1-\lambda}\right)}{1-\beta}$$

Under a privatised system with full insurance provided, for instance, by an asset market without any credit constraints, the lifetime expected utility is the Pareto first best is

$$V_{\text{insured}} = \frac{u\left(\lambda\theta_H + F(1, 1-\lambda)\right)}{1-\beta}$$

To calculate the welfare $V_{\text{constrained}}$ in the credit-constrained private property scenario, I numerically solve for the steady state general equilibrium policy functions $\left\{a_H^+(a), a_L^+(a)\right\}$, asset return r, and distribution of asset holdings $g(a)$. The steady state welfare measure is then

$$V_{\text{constrained}} = \frac{1}{1-\beta}\int_0^\infty g(a)\left\{\lambda u\left(c_H^P\right) + [1-\lambda]u\left(c_L^P\right)\right\} da$$

For a given value of r, the fixed point solving (4.5) is found through iterative optimisation of the two policy functions. The resulting asset distribution is then found by numerically inverting the policy functions and solving for $g(a)$ in (4.4), which takes the form

$$\lambda g\left(\tilde{a}_H(a^+)\right) = [1-\lambda]g\left(\tilde{a}_L(a^+)\right) \quad \forall a^+$$

[8] The relationship is

$$k = \frac{a\, r}{F_k\left(1, 1-\lambda\right)}$$

where the previously defined unit resource price is $p = F_k\left(1, 1-\lambda\right)/r$.

where $\tilde{a}_H(\cdot)$ is the inverse to the policy function $a_H^+(\cdot)$. The implied aggregate level of resource is then calculated as

$$K = \frac{r}{F_k(1, 1-\lambda)} \int_0^\infty a\, g(a)\, da$$

Since this level should be $K = 1$ for market clearing, the rate r is adjusted upwards or downwards using a bisection algorithm and the entire procedure is repeated with the new value of r.

High-precision convergence of this calculation proved elusive. The algorithm is plagued by a reliance on a large number of separate numerical optimisations, including nested optimisations and an inversion. As a result, the sensitivity of the final result, g, to the assumed value r and starting point was difficult to determine. The ability of the solver to converge was erratic and sensitive to the starting wealth distribution. Further refinements of the algorithm may be possible.

More or less arbitrary parameters were taken in order to test the model. These consisted of a Cobb-Douglas production function with resource share $\alpha = 0.36$, a period discount rate of $\beta = 0.9$, a good fortune parameter $\lambda = 0.7$, stochastic productivity levels $\theta_H = 2$, $\theta_L = 0$, and logarithmic period utility. The resulting resource sector wage is $w \approx 0.987$, considerably less than θ_H. Piecewise linear approximations were used for the policy functions. Because the steady state welfare is extremely sensitive to r, more than 20 iterations were needed in order to adequately constrain the result. Figure 5.1 on page 11 shows the calculated policy functions, value functions, and wealth distribution for the two-state private insurance scenario. Results from the equations above and from the general equilibrium calculation are:

$$\begin{aligned} V_{\text{insured}} &\approx 6.2206 \\ V_{\text{commons}} &\approx 6.1523 \\ V_{\text{constrained}} &\approx 6.154 \pm 0.005 \end{aligned}$$

To within calculable precision, the commons and the credit constrained private equilibrium offer the same overall expected welfare for the given parameters. It can be surmised that a number of other plausible parametrisations are likely to shift the comparison in either direction; for instance, increased risk aversion may make the privatised distribution less desirable.

6. Discussion

As mentioned above, the effect of aggregate shocks on insurance properties of the two regimes may be important. Because in the two-state model of Section 5 total output of the resource sector is assumed to be fixed by the parameter λ, stochastic fluctuations in λ would not lead to worse further over-exploitation of the CPR as compared with privately managed ownership. Under privatisation, however, fluctuations in λ would affect wage in the resource sector and would thus have a larger welfare impact on the poor

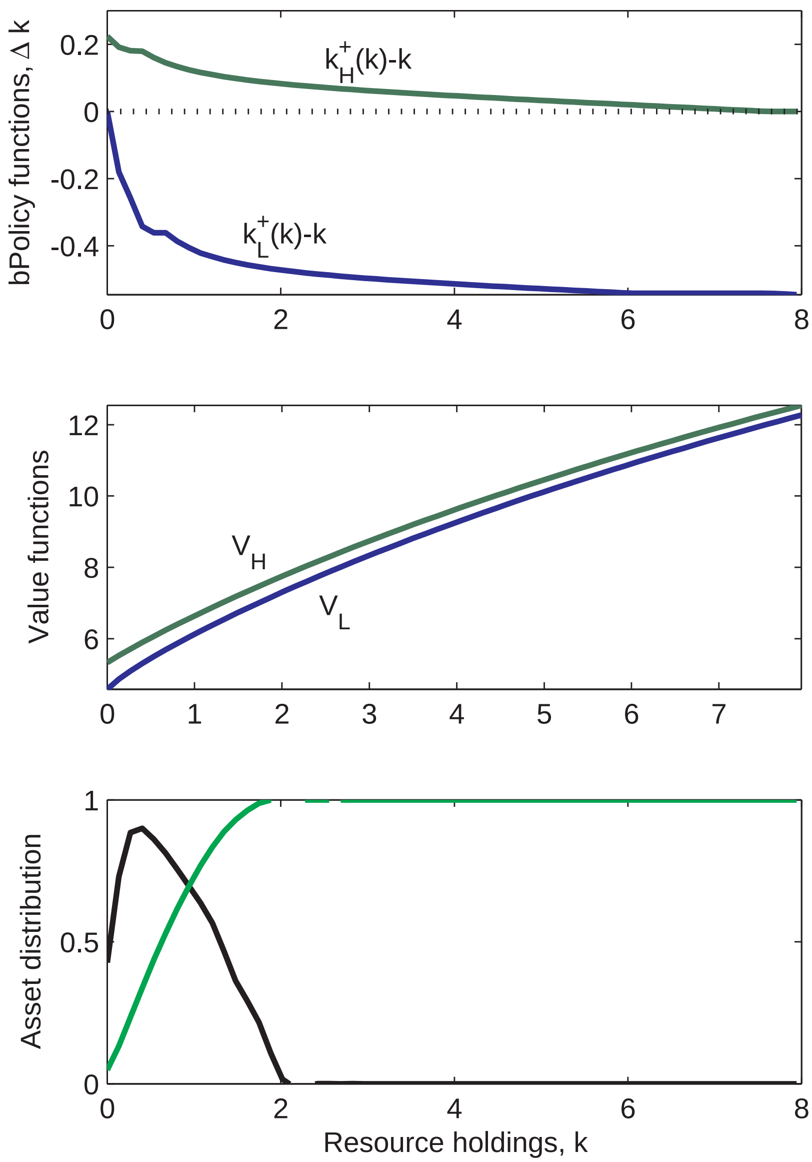

FIGURE 5.1. Results of the numerical solution. Top: policy functions for the two-state private scenario. Middle: value functions. Bottom: wealth distribution, density and cumulative.

end of the wealth distribution than on the wealthy end, thereby reducing somewhat the opportunity for insurance. A further analysis of this issue is warranted.

Quadrini and Rios-Rull [1997] investigate several modifications to neoclassical growth models with uninsurable idiosyncratic shocks to earnings in order better to reproduce measures of U.S. inequality. Among these is the suggestion that higher rates of return are earned by those with high asset levels. Indeed, the observation in the U.S.A. that wealthier households hold relatively high risk and high return assets while the poor hold liquid, safe assets is taken, along with high observed trading volumes and with the assumption of homogeneous risk aversions, as evidence of uninsured individual risk. In the model treated here, some form of economy of scale in production in the resource sector would result in a similar disparity. Nevertheless, an inequality in asset returns under full information would be incorporated rationally into the equilibrium price and hence into the distribution of asset holdings. Such a shift can be expected also to involve a reduction in the insurance value of the privatised regime.

Similarly, taking account of market power held by (wealthy) buyers over (poor) sellers in the resource asset market will shift downwards the equilibrium price, alter the skewness of the wealth distribution, and reduce the available insurance. Note, again, that this is a general equilibrium effect: under perfect information, any market power held by the wealthy over the poor is known to all and therefore shifts the equilibrium strategies and entire distribution of wealth, rather than driving a simple wedge between the welfare of the poor and the wealthy.

Extending welfare calculations to reflect the possible transition paths from some initial asset distribution towards the steady state distribution would complicate the analysis by introducing a time-varying asset distribution. An intermediate calculation would be to include some non-zero persistence (autocorrelation) in individual productivity shocks. This latter generalisation could be carried out in a steady state (static wealth distribution) framework. Intuitively, inclusion of such persistence would tend to give a welfare advantage to the commons insurance scheme, since the latter is better able to provide persistent subsidies to the poor.

Nevertheless, the welfare implications of the transition path following a privatisation change appear to be ambiguous. As compared with a privatisation scheme which initially assigns individuals randomly to wealth levels in accordance with the steady state distribution, a scheme with transition dynamics implies more persistent consumption shocks. This is because the consumption-smoothing insurance scheme considered here — precautionary saving — effects a persistence of shocks to consumption. As a result, the transition path offers less insurance than random assignment to the steady-state distribution. On the other hand, risk-averse individuals with a high enough rate of discounting of the future will prefer the transition path to the steady state if the transition starts off with an equitable (highly insured) initial distribution following privatisation. Thus, depending on the degree of risk aversion and temporal discounting, the omission of the transition path may bias my results in either direction. In practice, the transition path may indeed be dominant in determining society's welfare assessment and preferences over policy. Highly impatient individuals will not care about the eventual disparity in wealth implied by privatisation. The short term rewards (in the form of near-term insurance) of private redistribution may motivate privatisation even though it is a less efficient management system for future generations.

The model treated above lies within a standard framework in the neoclassical tradition. Certain more realistic modifications are worthy of discussion. Foremost is the consideration of psychologically plausible time discounting. Under hyperbolic discounting, agents overreact to short-term conditions, possibly causing later regret; that is, their decision making objective function is assumed to be different from the welfare used for normative evaluation. The phenomenon of under-priced sale of assets by those enduring a negative income shock, mentioned above, is known as "distressed sales" and may also be affected by hyperbolic discounting. In general, commitment devices are valuable to such agents when they are sophisticated enough to understand their own time-inconsistency. In this context, the ability to commit not to sell one's assets in times of poverty may be valuable. This form of commitment may exactly represent the advantage of communal ownership over private ownership in the face of individual variation in income.

This compelling conjecture is not treated in the present work. In principle, however, hyperbolic discounting might be detectable through an empirical analysis of labour choices and distributions of wealth both prior to and following the privatisation of a commons.

While the discussion and model presented so far have been focused on the context of locally managed rural resources, especially in developing countries, broader implications are possible, in particular from the consideration of psychologically realistic discounting behaviour. With "aware" or "sophisticated" hyperbolic discounters, even in advanced economies the value of socialised services such as health care may be derived in large part by the opportunity for commitment not to price oneself out of the service when luck — as measured by health and wealth — is low. While empirical literature highlights the fluctuations in individuals' income and consumption over time scales of only a year or two [Carroll, 2000], private insurance markets for health care and other social services do not allow commitment to service levels on comparable or longer time scales.

7. Conclusion

Understanding the conditions under which realistic private management of broadly used resources should be preferable to communal ownership is an important and still unfulfilled challenge for economics. An attempt to expand the scope of this understanding by considering the limited availability of insurance through asset markets did not lead to analytic inference. Although the range of steady state wealth levels may be much larger under private ownership as compared with a commons, sensible measures of welfare do not necessarily indicate that such dispersion is unfavourable. A numerical solution to a simplified comparison has so far generated inconclusive evidence. The implications of several possible variations on the model were discussed, with the suggestion that realistic causes of wealth dispersion and distressed sales under debt constraints are likely to favour collective insurance schemes.

Appendix: Two-state case

I sought a closed form solution for the simplest case of quantised savings choices. Consider the extra constraint that only two levels of resource assets, k_L and k_H, may be held. This adds an upper bound to the lower bound already implicit in the credit constraint, and discretises the savings decision. In addition, consider a discrete distribution over individual one-period productivity realisations, as in Section 5. In particular, suppose $f(\theta) = 0$ for $\theta \notin \{\theta_L, \theta_H\}$ and that the probability that $\theta = \theta_H$ is λ.

A steady state equilibrium for a given probability distribution λ is a static price p per unit of resource, a static resource rent rate r, a wage w for hired work on the resource, a consumption policy $c(\theta, k)$, a saving policy, and a distribution of resource holdings $\{g_L, g_H\}$ all subject to conditions analogous to those described previously. Because the individuals' choices are discrete (either to buy or not if $k = k_L$, or to sell or not if $k = k_H$), strategies may be mixed in equilibrium. Assuming $k^+(\theta, k) = k$ when $(\theta, k) \in \{(\theta_L, k_L), (\theta_H, k_H)\}$, let the mixed strategy be defined by the probability b of buying for $(\theta, k) = (\theta_H, k_L)$ and the probability s of selling when $(\theta, k) = (\theta_L, k_H)$. I also assume that $\theta_L < w$ in order that individuals enter the wage market when $\theta = \theta_L$. Equilibrium strategies satisfy the following conditions:

- given prices $\{p, r, w\}$, the policies $c(\theta, k)$, s, and b are optimal for the individual with preferences $\{u(\cdot), \beta\}$. That is, $s > 0$:

  $$u\left(w + rpk_H + p\left[k_H - k_L\right]\right) + \beta V_L \geq u\left(w + rpk_H\right) + \beta V_H$$

  and $b > 0$:

  $$u\left(\theta_H + rpk_L - p\left[k_H - k_L\right]\right) + \beta V_H \geq u\left(\theta_H + rpk_L\right) + \beta V_L$$

  where

  $$\begin{aligned} V_H &= \lambda\left\{u\left(\theta_H + rpk_H\right) + \beta V_H\right\} \\ &\quad + [1-\lambda]\left\{s\left[u\left(w + rpk_H + p\left[k_H - k_L\right]\right) + \beta V_L\right]\right. \\ &\quad \left. + [1-s]\left[u\left(w + rpk_H\right) + \beta V_H\right]\right\} \end{aligned}$$

  and

  $$\begin{aligned} V_L &= [1-\lambda]\left\{u\left(w + rpk_L\right) + \beta V_L\right\} \\ &\quad + \lambda\left\{b\left[u\left(\theta_H + rpk_L - p\left[k_H - k_L\right]\right) + \beta V_H\right]\right. \\ &\quad \left. + [1-b]\left[u\left(\theta_H + rpk_L\right) + \beta V_L\right]\right\} \end{aligned}$$

- land and labour on the resource are rented competitively: $w = F_L(K, L)$ and $r = F_K(K, L)/p$
- markets for labour and resource rental clear: $L = 1 - \lambda$ and $K = k_H G + k_L[1 - G] = 1$
- markets for trade in the resource clear: $\lambda b g_L = [1 - \lambda] s g_H$
- markets for trade in output clear (this will follow from Walras' law)

- and the distribution of wealth is constant

$$\begin{aligned} g_H &= g_H\left[\lambda + [1-\lambda][1-s]\right] + g_L \lambda b \\ &= g_H\left[1 - s + \lambda s\right] + g_L \lambda b \end{aligned}$$

  and similarly for g_L. In this simple case, these conditions for the distribution reduce identically to that for market clearing in resource trade, above.

For $k_L = 0$ and the pure strategy case $b = 1$, $s = 1$, I can reduce the overall *ex ante* value function for the private management steady state to one very complicated expression in parameters and p, which makes numerically solving for equilibria relatively easy. I could not solve for p in this case or in the cases in which one of $\{b, s\}$ is less than 1. For the pure strategy case, solutions exist (as ascertained numerically) for a continuum of prices over a finite range. However, drawing conclusions from this model would be awkward, since the choice of the two asset levels available affects welfare. Overall, this two-wealth situation is not of much interest, as it is too restrictive and still appeared to be analytically intractable.

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